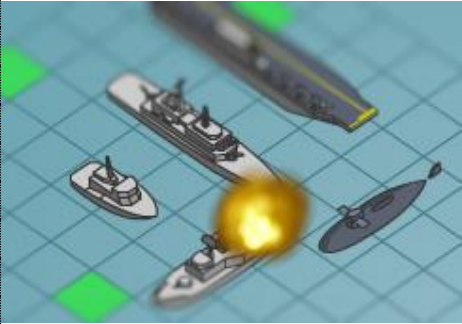


14. Graphs and functions

BATTLESHIPS

YOUR SHIPS

					5					
					4					
					3					
					2					
					1					
-5	-4	-3	-2	-1	0	1	2	3	4	5
					-1					
					-2					
					-3					
					-4					
					-5					



The illustration shows a 10x10 grid with a battleship, a cruiser, a destroyer, and a submarine. A yellow explosion indicates a hit on the battleship at coordinate (2, 2).

					5					
					4					
					3					
					2					
					1					
-5	-4	-3	-2	-1	0	1	2	3	4	5
					-1					
					-2					
					-3					
					-4					
					-5					

FLEET

#	SHIP	SIZE
1x	Aircraft Carrier	5
1x	Battle ship	4
1x	Cruiser	3
2x	Destroyer	2
2x	Submarine	2

ENEMY SHIPS

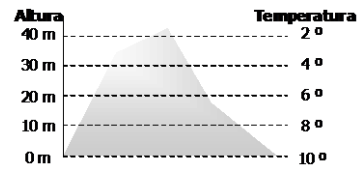
RULES:

- The game is played on four grids, two for each player.
The grids are typically square – usually 10×10 –.
On one grid the player arranges ships and records the shots by the opponent.
On the other grid the player records his own shots and his reached targets.
Our grid is a math grid. You can use the grey squares. Notice these squares have a zero coordinate.
- Before the game begins, each player secretly arranges their ships on their first grid.
Each ship occupies a number of consecutive squares on the grid, arranged either horizontally or vertically.
The number of squares for each ship is determined by the type of the ship.
The ships cannot overlap.
- After the ships have been positioned, the game proceeds in a series of rounds.
In each round, each player's turn consists of announcing a target square in the opponent's grid which is shot.
If a ship occupies the square, then it takes a hit.
The opponent has to say 'hit'. In the opposite way he or she will say 'water'.
- The opponent then fires a shot at the first player's ships.
When all of the squares of a ship have been hit, the ship is sunk.
When a player has all the ships sunk then the game ends.

1. INFORMATION PROCESSING

Science is based on the study of information we collect from a specific event in order to discover relationships.

To represent the information, we use two methods: tables and graphs.

**2. TABLES**

A table is a method to write the related values in an experiment or event.

We use two rows or two columns with the related pair values.

Example:

Complete the information of the picture in the table.

High (m)	Temperature (°C)
0	10

3. CARTESIAN COORDINATES

To represent information with two variables we use two numerical perpendicular straight lines with a common origin.

They are the coordinate axes.

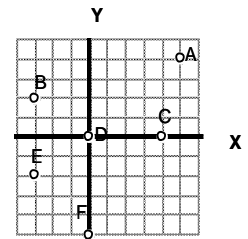
The horizontal is called abscissa axis and we write X.

We represent the first variable on it.

The vertical axis is called ordinate axis and we write Y.

We represent the second variable on it.

Each pair of values is represented by a point whose coordinates are (x, y).



Example:

Write the coordinates for each point you see in the graph:

A (5, 4)

D

B

E

C

F

4. GRAPHIC CHARACTERISTICS

A graph is increasing if y grows when x also grows.

A graph is decreasing in the opposite way.

A maximum is a point in a graph where the graph is increasing on the left and decreasing on the right.

A minimum is a point where it is the opposite.

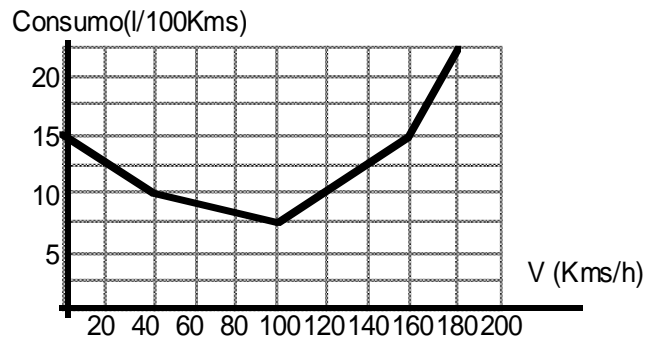
A graph is discrete if there are isolated values only.

A graph is continuous if it is represented by a continuous line.

Example

This graph shows the petrol consumption of a car according to its speed:

- What is the first variable?
- What are the units of each axis?
- What is the consumption to forty kilometres an hour?
- What is the speed with the least consumption?
- At what speed the consumption is 10 litres also?
- Write the decrease and increase intervals.

**5. LINEAR FUNCTION****Direct proportionality**

It is the relationship between two magnitudes which are in direct proportion.

For example, we buy oranges €0.5 kilo.

Write some related values, for instance, the prices we pay for 1 kg; 2 kg; 3 kg...

$$x = 1 \text{ kg}; y = 0.5 \cdot 1 = \text{€}0.5$$

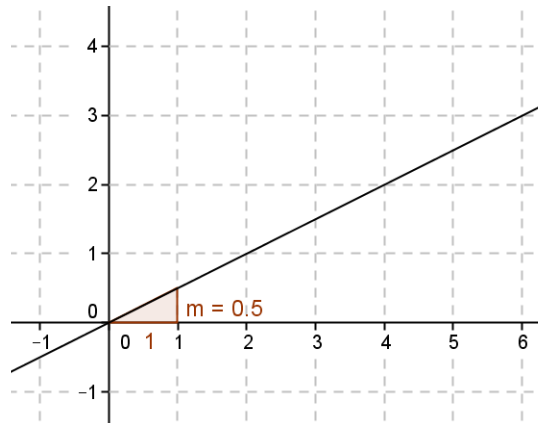
$$x = 2 \text{ kg};$$

$$x = 3 \text{ kg};$$

Graph

The graph for a linear function is a straight line through the origin of coordinates.

This is the graph for $y = 0.5x$



The constant of proportionality is the slope of the straight line.

This is a measure of its inclination. Here it is $m = 0.5$

The slope is the ratio between whatever related values 'y' and 'x' because it is a proportion; and we represent it by 'm': $m = \frac{y}{x}$

The slope is 0.5 in the example.

Equation or formula

The equation or formula is simple: $y = m x$.

'm' is the slope or gradient. It is the constant of proportionality.

$m > 0$ means the function is increasing.

$m < 0$ means a decreasing function.

Write the formula that relates the weight and the price in the example.

Function from one point

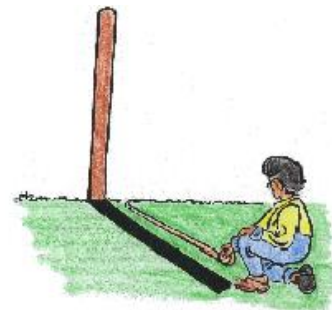
One point or one data is enough information to get the formula for a direct proportion.

If I know how much a weight of oranges is, I will know the formula immediately.

Let's say $A(x, y)$ is whatever point or data from the function (chart).

We get the constant of proportionality in this way:

$$k = \frac{y}{x}$$



Example

You measure at 12 o'clock the shadow of a post 4 m long –x–.

If the shadow length is 8 m –y– could you deduce the formula that relates the length and the shadow at this hour?

Affine function

Definition

This is the kind of relation between two proportional variables when the initial value is different from zero.

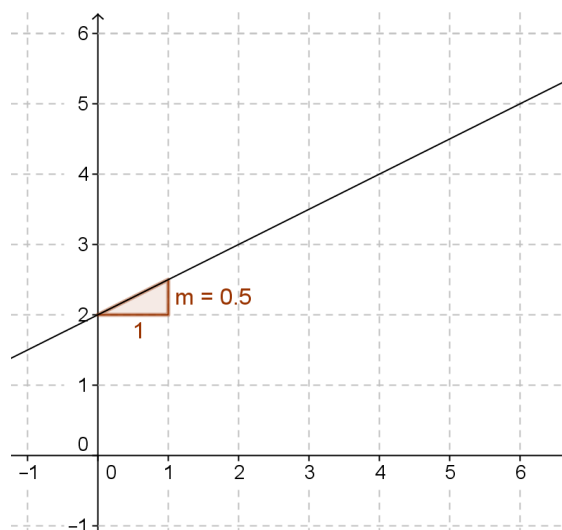
For example, the price of a taxi trip is proportional to the distance travelled but at the same time, it has an initial quantity added.

The formula is a first-degree function, that is, $y = mx + b$.
Now 'm' is the constant of proportionality and b is the initial value.

Example:

The price of a taxi is €2 –fixed quantity- plus €0.5 per kilometre. What is the formula for the price of a trip?

The graph is a straight line whose slope is 'm', 0.5 in the example, and whose ordinate in the origin is 'b'; 2 in the example.



'm' value is called the slope or gradient and indicates how steep the line is. 'b' value is called the y-intercept, the point that the line cuts the y-axis.

Function from two points

Two points or data are enough information to get the formula for an affine function.
Let's say $A(x_1, y_1)$ and $B(x_2, y_2)$ are two whatever points from the function (chart).

We get the slope thus: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

After that, it is easy to work out 'b' from any point A or B by replacing the previous value into the general formula $y = mx + b$ and clearing 'b'.

Example

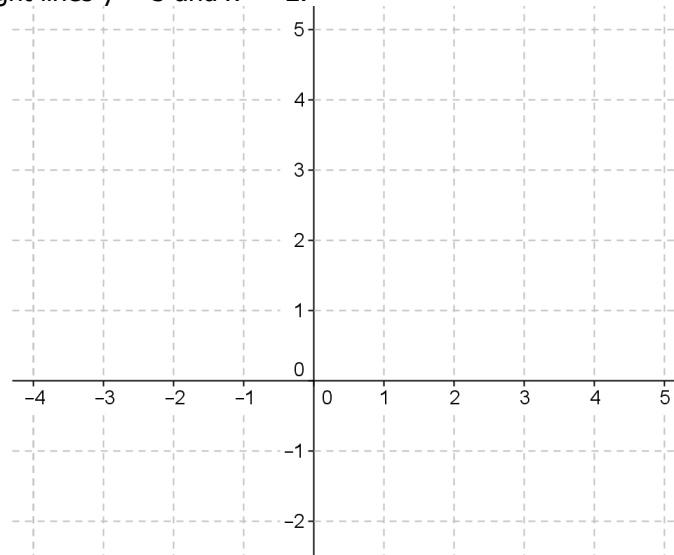
Work out the formula of an affine function that has (2, 7) and (3, 9) as values.

Horizontal and vertical lines

A horizontal line has a simple equation: $y = b$; that is, its slope is 0. It is a constant function.

A vertical line is simple also: $x = a$; that is, 'y' could be whatever value but 'x' always has the same value 'a'.

Draw the straight lines $y = 3$ and $x = -2$.



EXERCISES AND PROBLEMS

1. Information processing

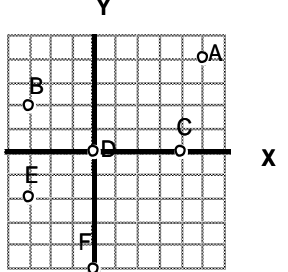
2. Tables

1. A person weighing 60 Kg burns 2 Kcal in 5 minutes.
 Make a table that relates both variables taking the following values of X: 5, 10, 15, 20 and 25 minutes.
 Make a graph that shows this information.

3. Cartesian coordinates

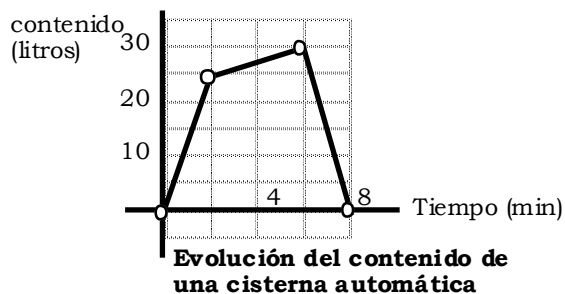
2. Draw a Cartesian diagram with the following points: A(-1, 0); B(0, 1); C(1, 0); D(3, 0); E(2, -1); F(1, -1); G(0, -2); H(0, 0)
 Join the points in order to create a picture.

3. What's the name of the X-axis?
 There are some points on the diagram. What are their coordinates? A (,)...

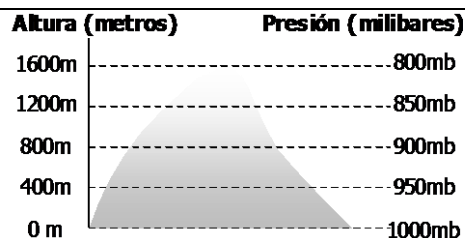


4. Graphic characteristics

4. In this graph:
 a) Write the variables and its units.
 b) Make a table with the points that you see in the graph.
 c) Which is the maximum value?
 d) What are the increasing interval and the decreasing interval?
 e) Write the process that describes this graph.



5. You can see the evolution of the pressure by the height in this graph.
 a) What are the variables and the units of them?
 b) Write in a table the associated values.



6. What is the order of the X-axis?
 In this graph you can see the relation between the height and the weight of some people.
 Who is the tallest?
 Who is the shortest?
 Who is the heaviest?
 Who is the lightest?
 Order people you can see in the

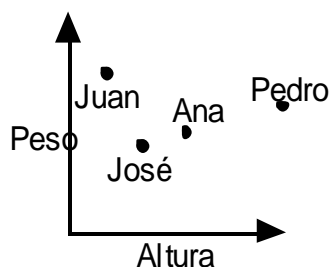
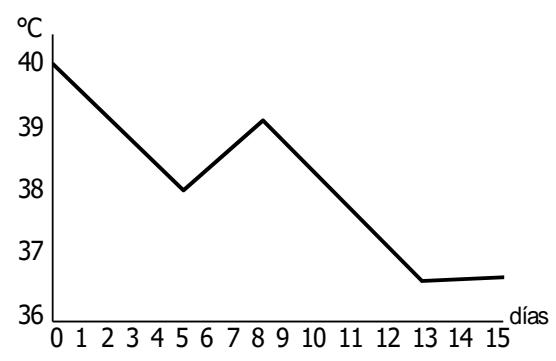
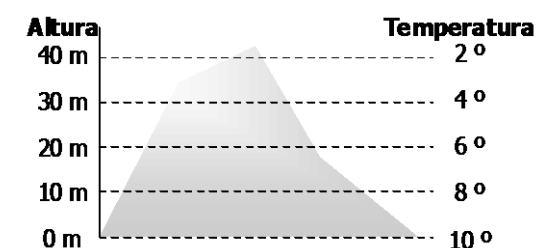
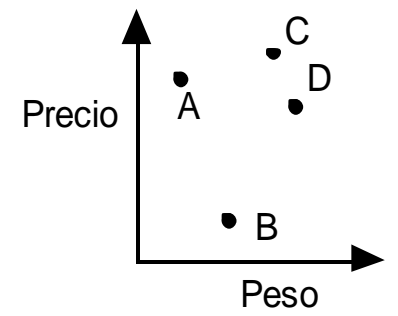


chart according to their height.	
<p>7. What is a decrease function?</p> <p>The graph shows the evolution of the temperature of a patient in fifteen days. Knowing that the normal temperature is 36.5° answer the following questions:</p> <ol style="list-style-type: none"> What are the units for each variable? How many days was the patient sick? When is the highest temperature and when is the lowest temperature, approximately? Write the increasing intervals and the decreasing intervals. 	
<p>8. You can see the information about the evolution of the temperature by the height in this graph.</p> <ol style="list-style-type: none"> Which are the variables and which are the units? Make a table with the linked values. Make a graph with two axes. 	
<p>9. What's the name of the intersecting point of the two axes of coordinates?</p> <p>We can see the price and the weight of several products of a supermarket in the next graph.</p> <p>Identify each of them by the data you have below.</p> <ul style="list-style-type: none"> The most expensive is the sausage. The heaviest are the beans. The cheapest is the pasta. The lightest is the tuna. 	

5. Linear function

- 10.** a) Complete the next chart in order to get a direct proportionality:

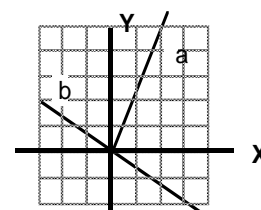
x	2	3	5	7
y			12	

- b) What is the constant of proportionality?
- c) What is the linear function formula?

- 11.** A person weight 60 kg and burns 2kcal per 5 minutes.
- Do a table that relates both variables. Write 5 values at least.
 - Draw the graph.

- 12.** Find the slope of the following lines
- What is the formula for the functions they represent?

- 13.** Make a chart of values (three values) and the corresponding graph for

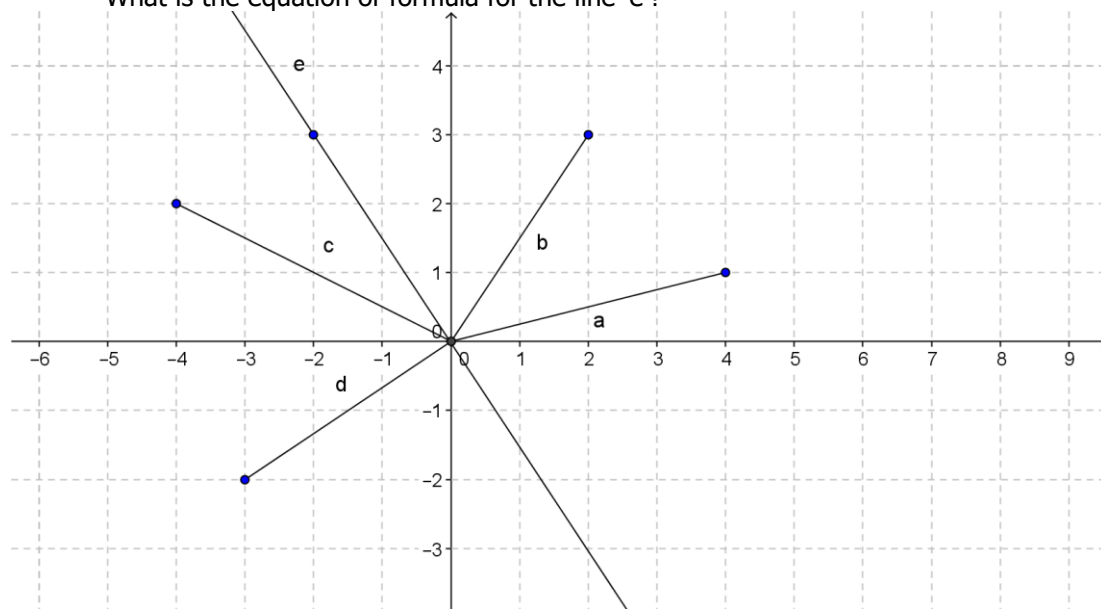


the following functions:

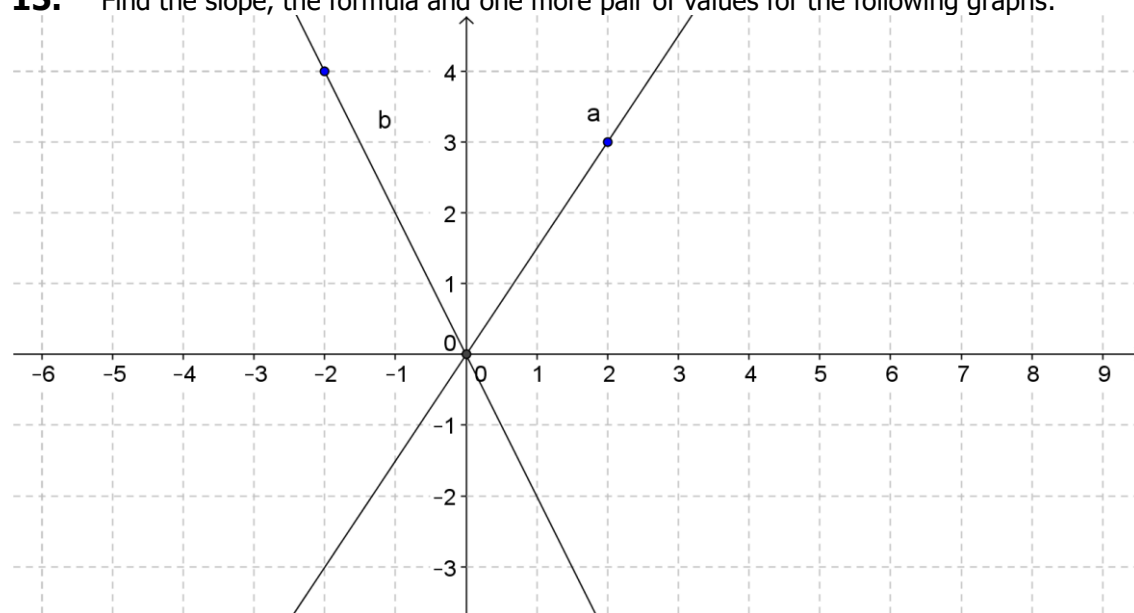
a. $y = 5/3x$

b. $y = -1/5x$

- 14.** Work out the slope for the following segments and straight line:
What is the equation of formula for the line 'e'?



- 15.** Find the slope, the formula and one more pair of values for the following graphs:



- 16.** Using the graph answer the following questions:

a) Make a table with the two values A and B.
 b) Calculate the slope.
 c) Work out the formula.

- 17.** Draw the graph of the function: $y = -3x + 5$

- 18.** The formula of a function is $y = -2x + 3$.

a. How much is its slope and its y-intercept?
 b. Do a table with 3 values.
 c. Draw the graph.

- 19.** Calculate the formula of a first degree function which takes the following values:

x	y
2	4
5	10

- 20.** Draw the line through points A (2, 1) and B (5, 7). Find the formula.

- 21.** Using the graph answer the following questions:

a) Do a table with the two values A and B.
 b) Calculate the slope.
 c) Work out the formula.

- 22.** Calculate the function formula of degree 1 which takes the following values:

x	y
2	5
4	8

- 23.** Draw the line through points: A(-3, 0) and B(4, -2). Find the formula.

- 24.** Draw the straight line through points: A(-3, 0) and B(4, -2). Find the formula.

- 25.** The formula of a function is $y = 2x - 6$.

a. How much is its gradient and its y-intercept?
 b. Do a table with 3 values.
 c. Draw the graph.

- 26.** Draw the straight line through points: A(-2, 3) and B(0, -2).

- 27.** Find the formula for the linear function that passes through the points: A(-2, -3) and B(6, 1). Draw the graph.

- 28.** Using the graph answer the following questions:

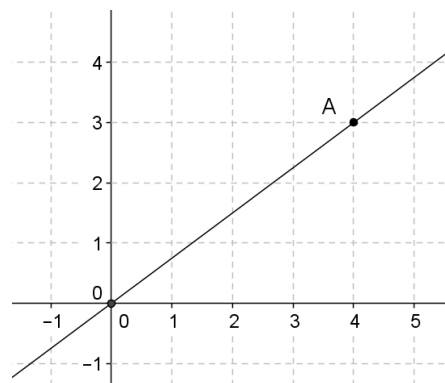
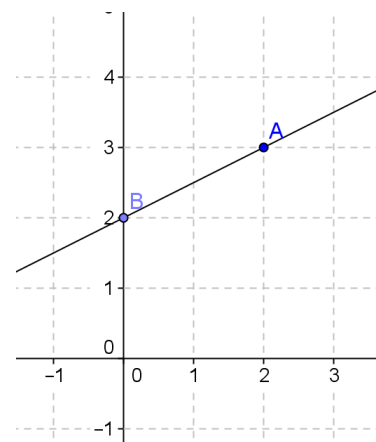
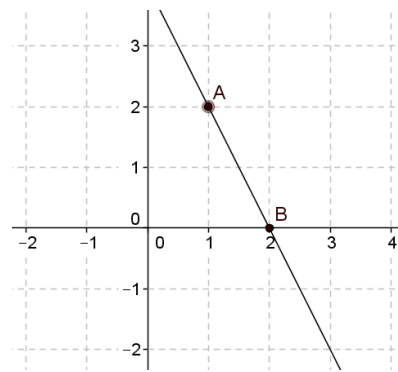
a) Do a table with two values.
 b) Calculate the slope.
 c) Work out the formula.

- 29.** Draw the straight line through points: A(3, -4) and B(0, 2).
 Find the formula.

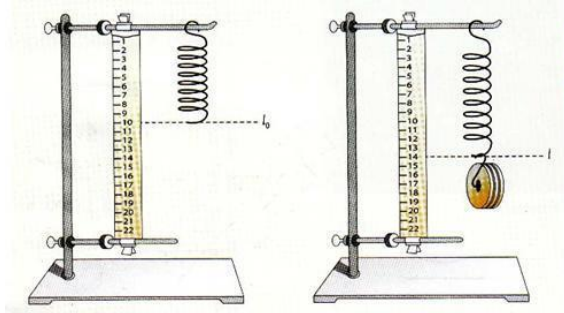
- 30.** The formula of a function is $y = 4x - 3$.

a. How much is its slope and its y-intercept?
 b. Do a table with 3 values.
 c. Draw the graph.

- 31.** The formula of a function is $y = -3 + 5x$.
 a. How much is its gradient and its y-intercept?



- b. Do a graph with 3 values.
c. Draw the graph.
- 32.** The formula of a function is $y = -2x + 3$.
a. How much is its slope and its y-intercept?
b. Do a table with 3 values.
c. Draw the graph.
- 33.** A 10 inch spring hangs from the ceiling. Its elongation is 4 inches per hanging kg. We study the function that establishes the relation between the total length of the spring according to the weight set.
a) Make a table with 3 values.
b) Find the function formula
c) Represent it.



- 34.** The price for a taxi is €1.5 plus €2 per kilometer. If we call x the number of kilometres travelled and y the tour price. What is the formula for the function?



- 35.** The price for a taxi is €2 and €3 per kilometer. If we call x the number of kilometres travelled and y the tour price.
a. Do a table with three values.
b. Draw a graph.
c. What is the formula for the function?
- 36.** A train ticket costs €2 as insurance plus 6 cents per km.
a. Write a table that relates the distance travelled with the price. Use 100 km as the unit of distance.
b. Draw a graph
c. Write the formula.
d. How much is a 1800 km trip?
- 37.** A repair costs €5 for travel expenses plus 12 € per hour of work. Find the formula for the function that relates the cost with the time spent on the repair.
- 38.** I paid €10 to repair my car for the flat fee plus €18 for every worked hour.
a. Make a table that relates the working hours with the price.
b. Make a graph.
c. Find the function formula.
d. How much will a repair be if it takes six hours and a half?
- 39.** I paid 3 euros for a 100 Km trip and 7 euros for a 200 Km trip. The price is the result of adding the insurance plus a quantity per kilometer.
a) Make a table with the two above values.
b) What is the price per km?
c) What is the insurance fee?
d) What is the formula that relates the price with the travelling distance?